Spatiotemporal Nonlinear Dynamics of a Magnetoelastic Ribbon

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ABSTRACT

Magnetoelastic materials have a strong coupling between strain and magnetization, so applying a magnetic field to a magnetoelastic material can change its shape. This coupling leads to interesting dynamics. We have studied the dynamics of a wide ribbon of Metglass 2605sc which was driven by a magnetic field. The ribbon was suspended as a pendulum in a set of Helmholtz coils, which provided both DC and AC magnetic fields. Laser light was reflected off the ribbon to measure its angular displacement. Two points on the ribbon could be simultaneously illuminated, and one of the laser beams could be scanned over the ribbon. We observed quasiperiodic bifurcations in the motion of the ribbon, and characterized the spatial aspect of the motion with some recently developed statistics.

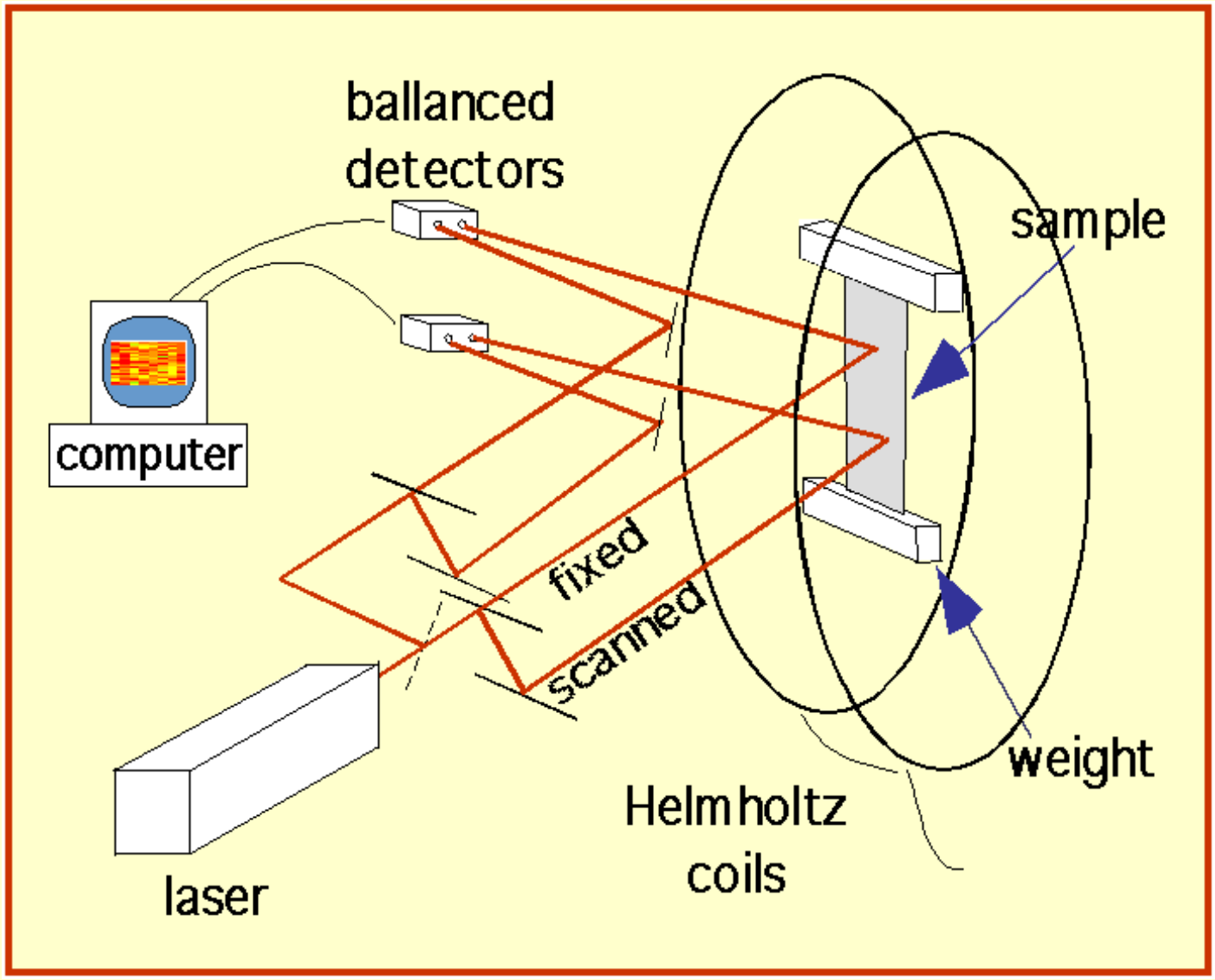
Motivation

- Simple quasi-2D system displaying complex spatio-temporal dynamics.
- Intuitively, simultaneous dynamics at two different sites should be functionally related.
- Poor linear correlation after first bifurcation.
- Illustrates the utility of Pecora's generalized functional statistic, Θ_{c0}.
  - Real experimental system with noise.
  - Linear Cross Correlation fails.

MATERIALS & GEOMETRY

- Materials
  - Magnetostrictive 2605sc Metglas Ribbon
  - Non-Magnetostrictive 2705m Metglas Ribbon (comparison)
  - As cast (not annealed)
  - AlliedSignal Inc.
- Geometry
  - Ribbon freely suspended (clamped on top - 1.6g weight on bottom)
  - AC + DC Magnetic field in ribbon plane (at rest) to long axis
    - H_{DC}: Amplitude = 6 Oe
    - H_{AC}: RMS Amplitude = 0 to ~ 6 Oe, Frequency 0 to < 10 KHz
      - (In this paper limited to bending modes: f_{ac} = 3 to 300 Hz)
  - Metglas ribbons ~ same dimensions (25 mm x 60 mm x 1 mil)

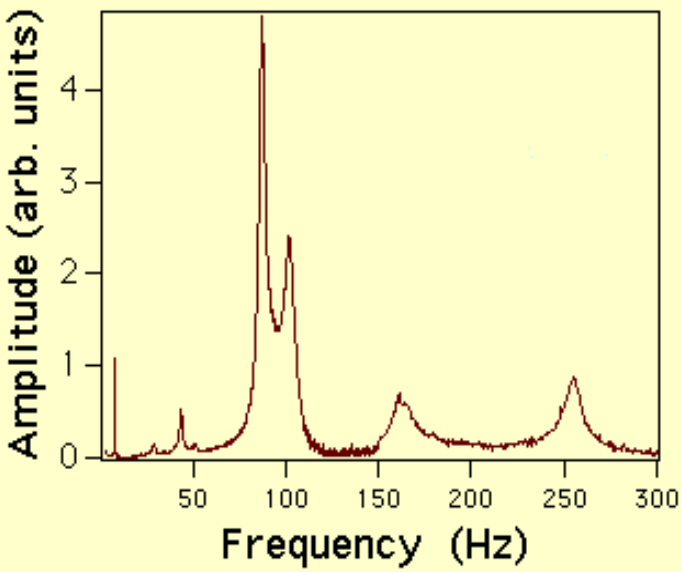
Experiment Sketch



Experimental

- Two sampes: One Magnetostrictive, one Non-Magnetostrictive
- Examine two modes near 114 and 244 Hz
- Time series: Strobed at driving (H_{ac}) frequency
- Embedding Dimension = 3
- Single spot
  - Detect deflection amplitude data
  - Mode spectra
  - Bifurcation diagrams
- Dual spot:
  - 1) Stationary reference at center of sample & 2) Scanned reflection
  - Detect deflection phase data:
    - Spatial amplitude varied from spot to spot across sample due to sample texture.
    - Spatial phase data comparable.

MODES

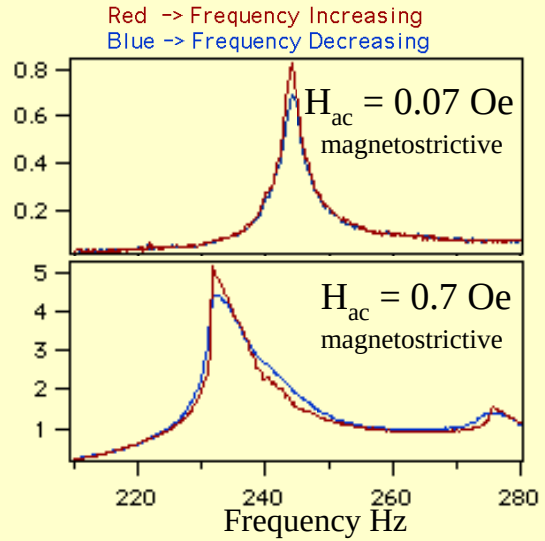


- Calculated modes
  - Freely suspended, Clamped & Weighted Plate
  - Two samples:
    - Same geometry
    - ≈ Same Young's Modulus
  - Calculations Experiment studied low order bending modes
- Energy:

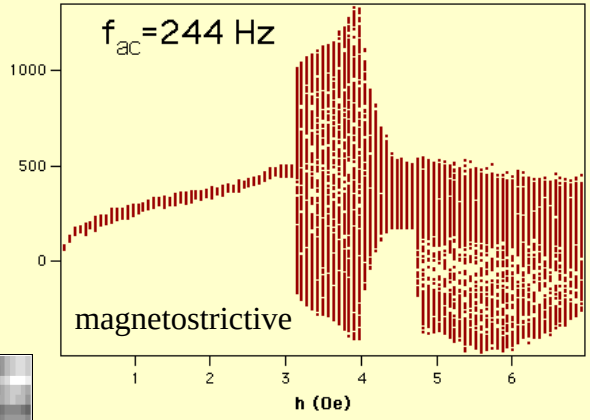
$$E_{Total} = -HM_s\alpha_y - K\alpha_x^2 - b\left(\varepsilon_{xx}\alpha_x^2 + \varepsilon_{yy}\alpha_y^2 + \varepsilon_{xy}\alpha_x\alpha_y\right)$$
$$+\frac{1}{2}C_{11}\varepsilon_{xx}^2 + \frac{1}{2}C_{11}\varepsilon_{yy}^2 + C_{12}\varepsilon_{xx}\varepsilon_{yy} + \frac{1}{2}\left(\frac{C_{11}-C_{12}}{2}\right)\varepsilon_{xy}^2 - \sigma\varepsilon_{yy}$$

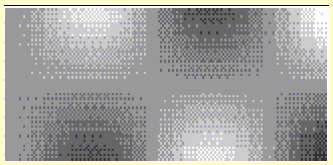
  - Where: M_s saturation magnetization, K anisotropy constant, c's elastic constants, ε's stresses, b magnetoelastic coupling constant, α's cosines from x and y axes.
  - Rayleigh-Ritz technique $w(x,y;t) = \sum_{m=0}^{M}\sum_{n=0}^{N} a_{mn}\Psi_m(x)\Phi_n(y)$
    - Where: $\Psi_m(x)$ & $\Phi_n(y)$ Euler-Bernoulli clamped-free & free-free beam functions

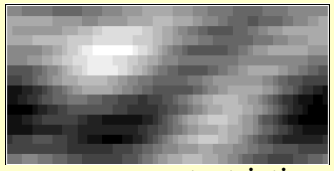
Non-Linear Effects

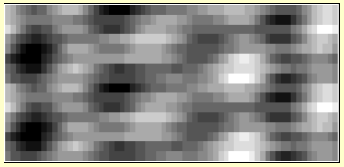


- Magnetostrictive Ribbon
  - Foldover and Hysteresis in deflection vs driving frequency
  - Quasiperiodic bifurcations in time series
  - Spatial modes significantly altered
    - Domain effects: At H_{dc} = 35 Oe the calculated, non-magnetostrictive and magnetostrictive spatial mode patterns are similar



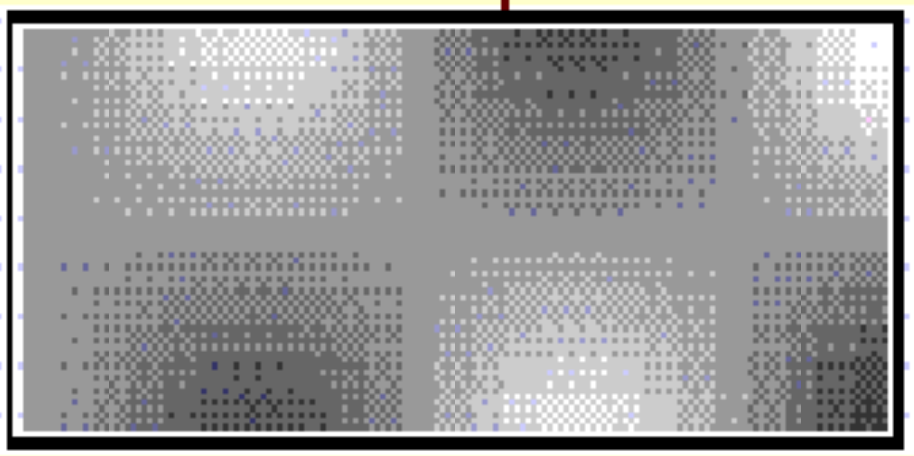

calculated


non-magnetostrictive

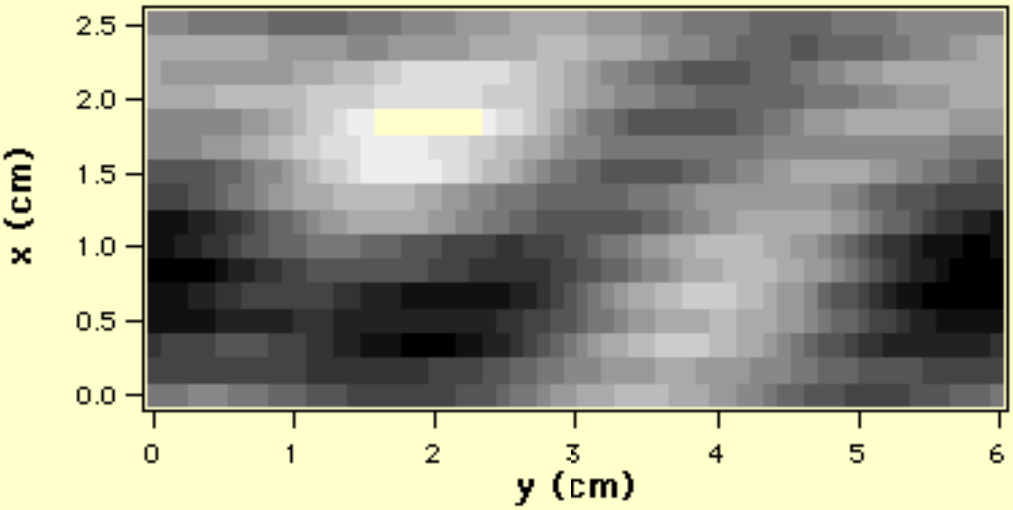

magnetostrictive

- Non-Magnetostrictive Ribbon
  - No non-linearities at these applied fields
  - Check simple mode theory

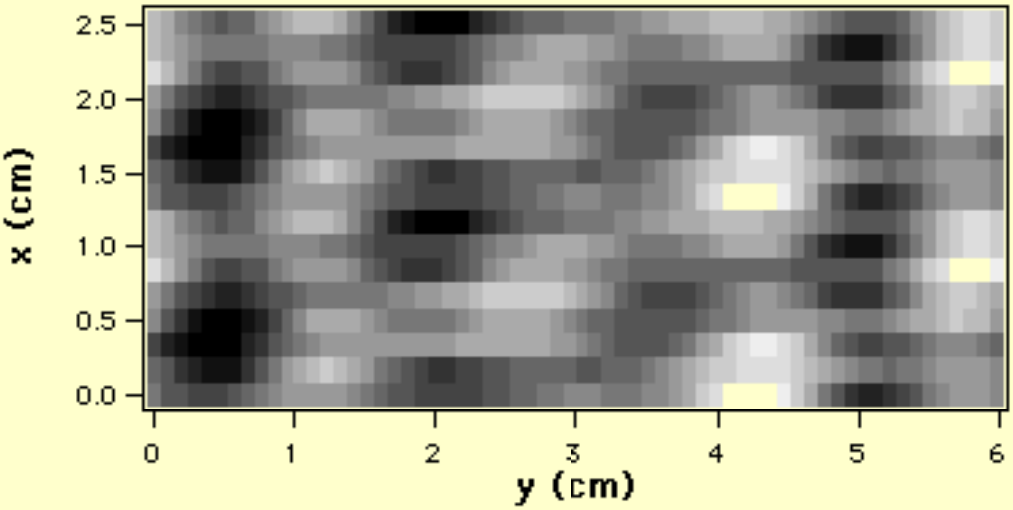
Mode Comparison 1



Calculated mode amplitude of a bending mode at 132 Hz.

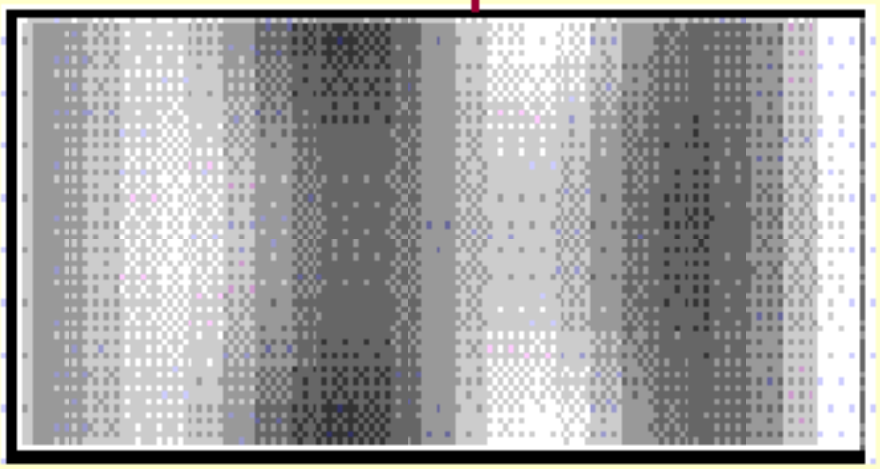


Experimentally measured phase response (filtered) of a mode at 129 Hz in a non-magnetostrictive 2705m ribbon.



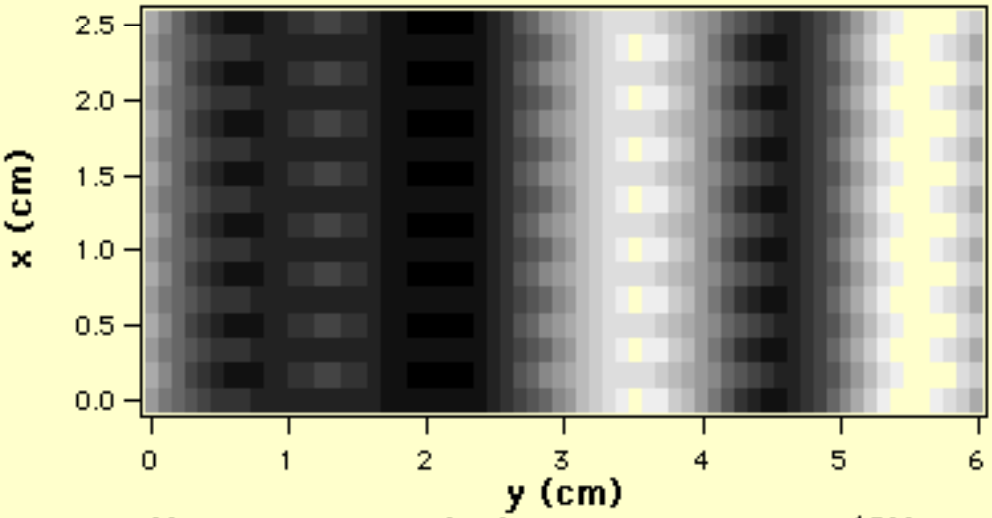
Experimentally measured phase response (filtered) of a mode at 114 Hz in a magnetostrictive 2605sc ribbon.

Mode Comparison 2



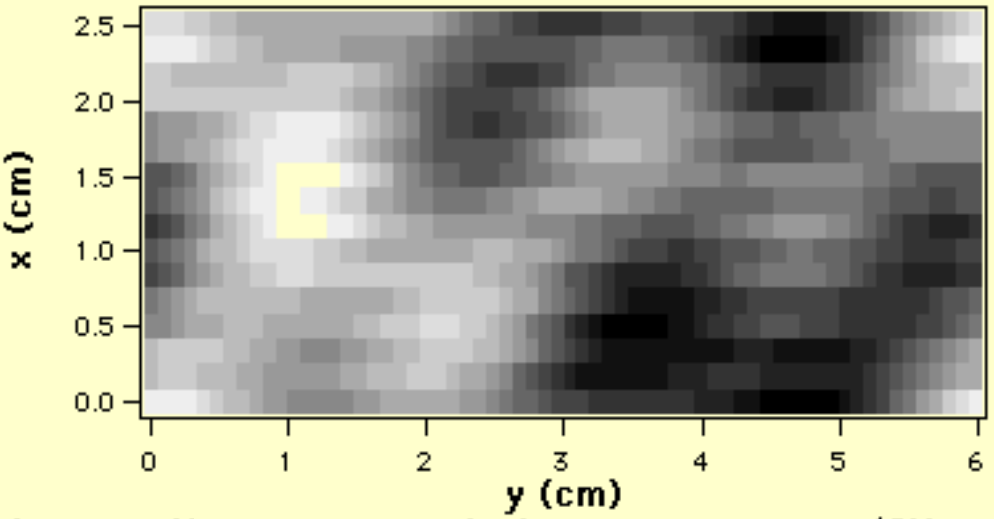
(a)

Calculated mode amplitude of a bending mode at 256 Hz.



(b)

Experimentally measured phase response (filtered)
of a mode at 256 Hz in a nonmagnetostrictive 2705m ribbon.



(c)

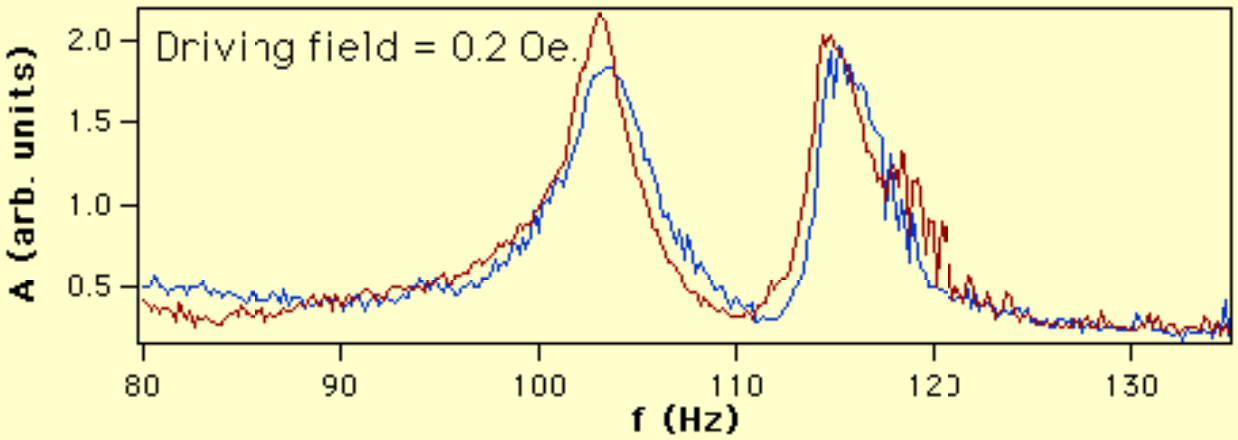
Experimentally measured phase response (filtered)
of a mode at 244 Hz in a magnetostrictive 2605sc ribbon.

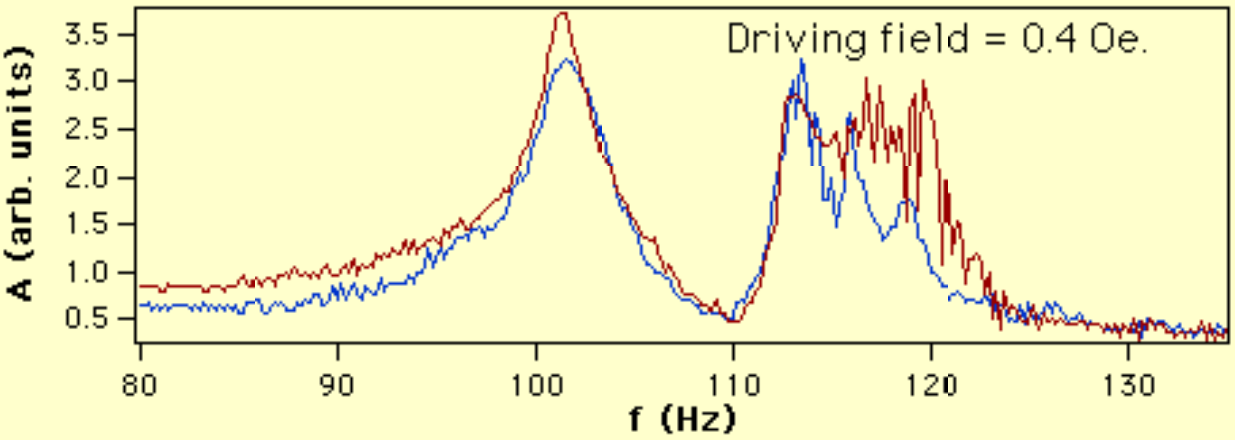
HYSTERESIS

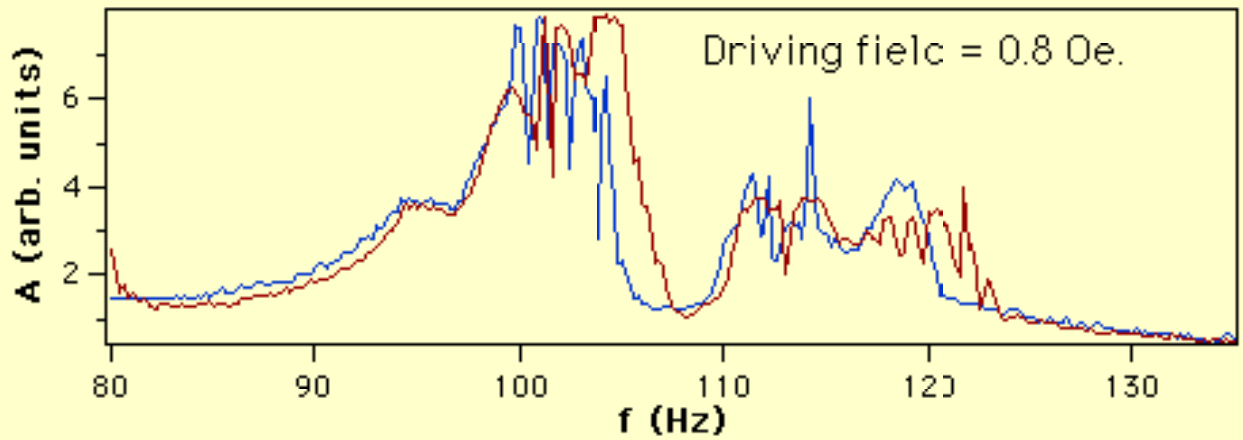
Magnetostrictive Ribbon

Red -> Frequency Increasing

Blue -> Frequency Decreasing







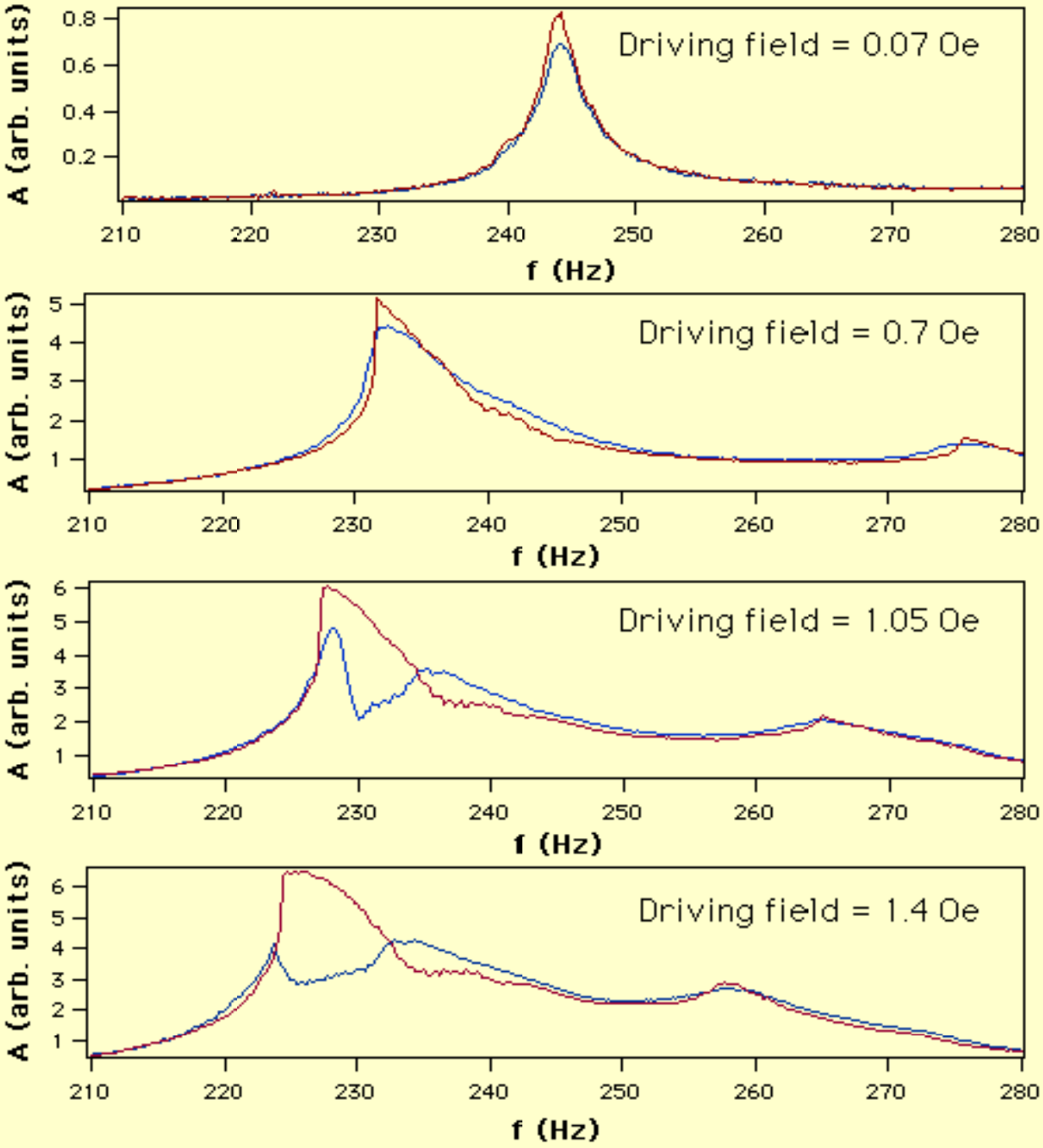
Amplitude spectrum for the magnetostrictive 2605sc ribbon between 80 and 130 Hz for3 different values of the driving magnetic field.

HYSTERESIS

Magnetostrictive Ribbon

Red -> Frequency Increasing

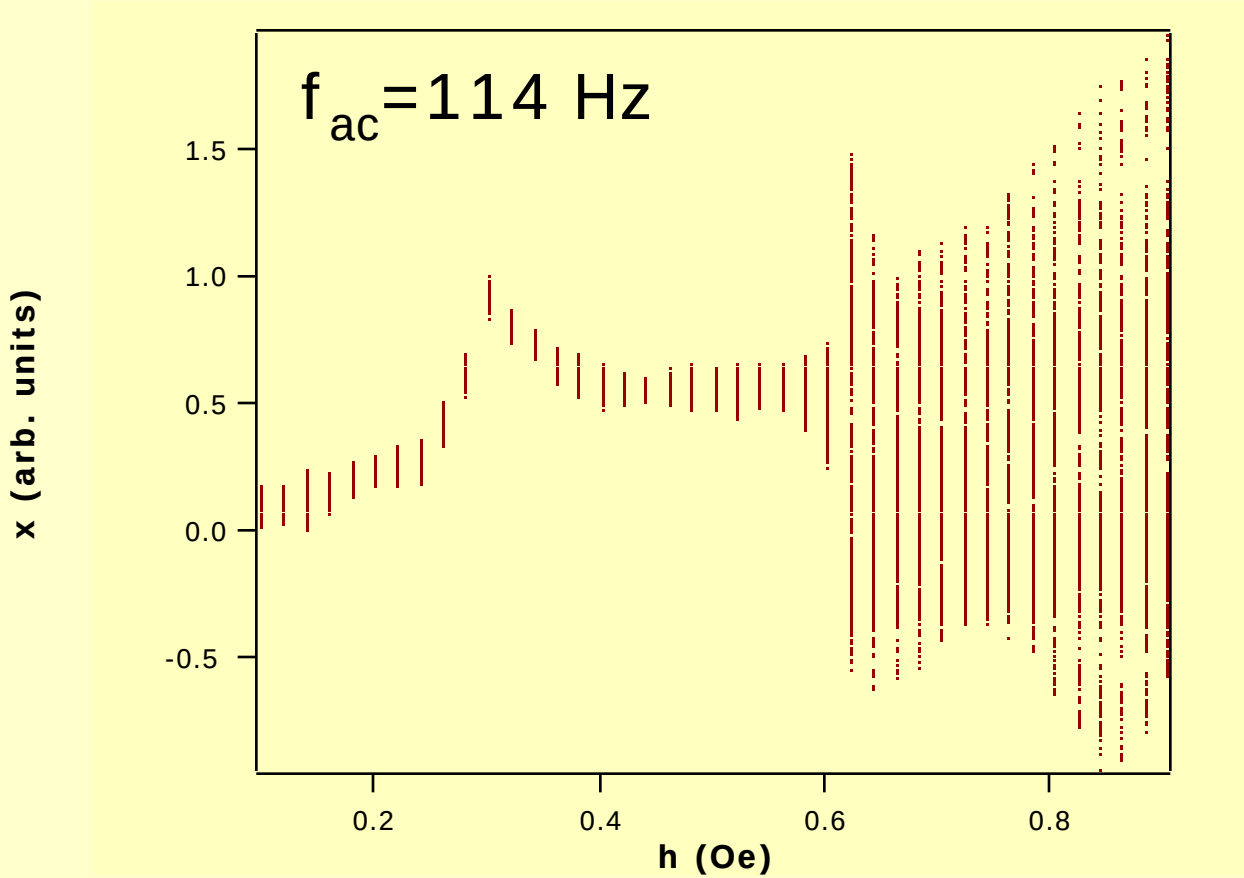
Blue -> Frequency Decreasing



Amplitude spectrum for the magnetostrictive 2605sc ribbon between 210 and 280 Hz for 4 different values of the driving magnetic field.

Bifurcation diagram

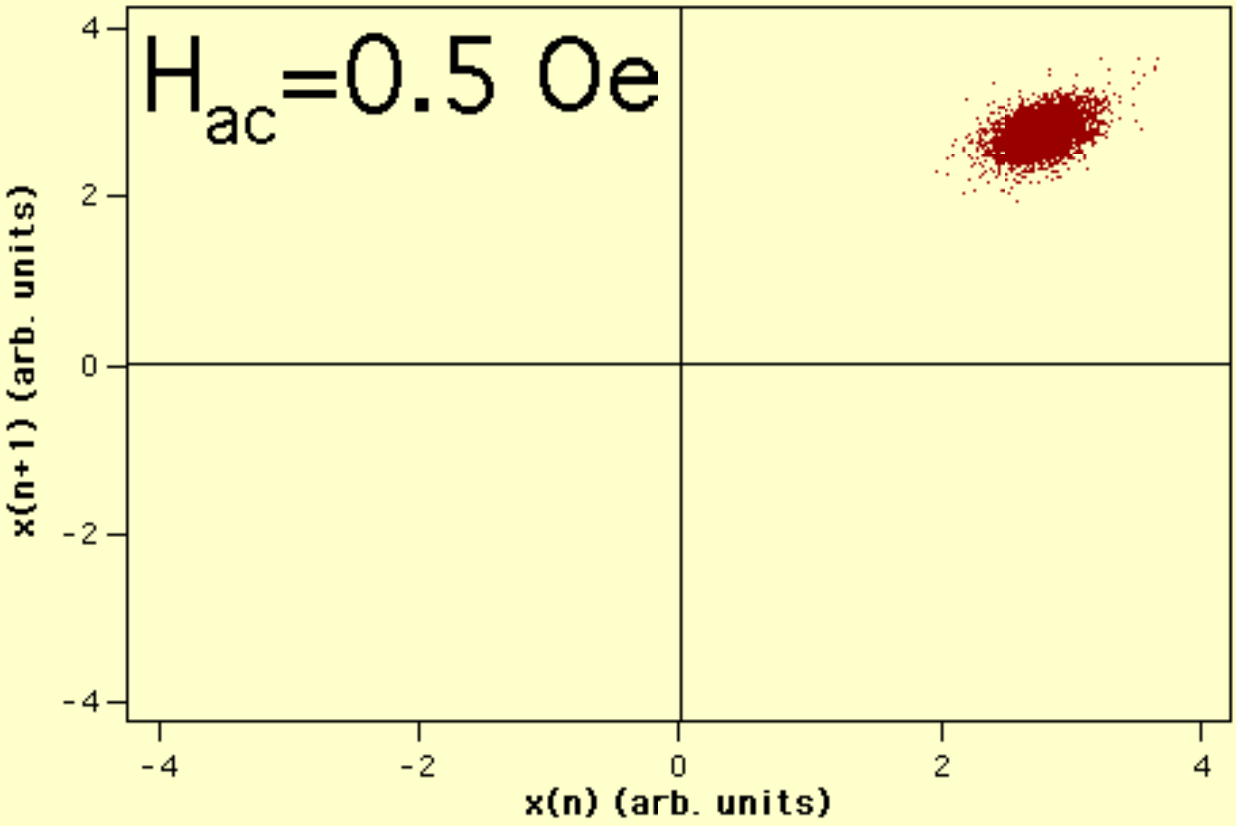
Magnetostrictive 2605sc ribbon
Driven at 114 Hz.

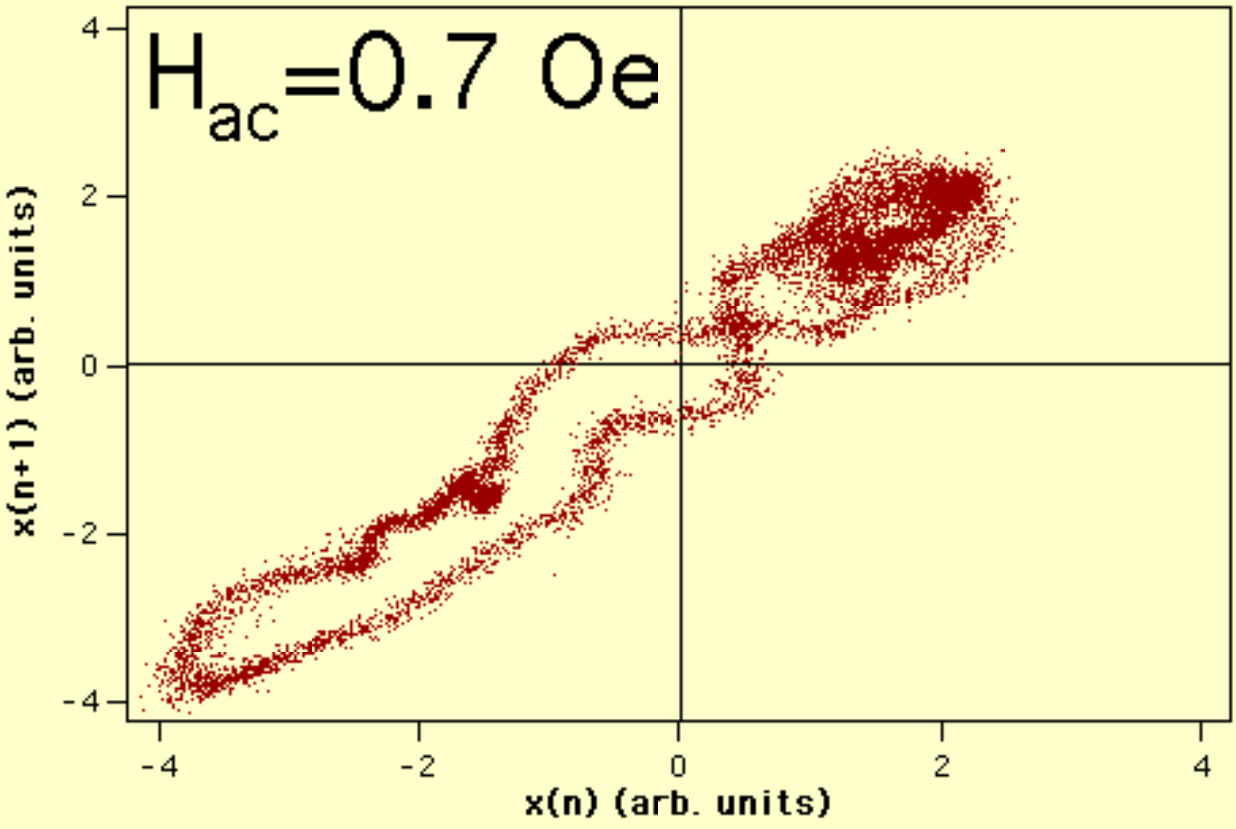


The peak-to-peak magnetic driving field is h, while x is the value of the detector output when the driving signal crosses zero in the positive direction.

Poincare Sections

magnetostrictive ribbon driven at 114 Hz.

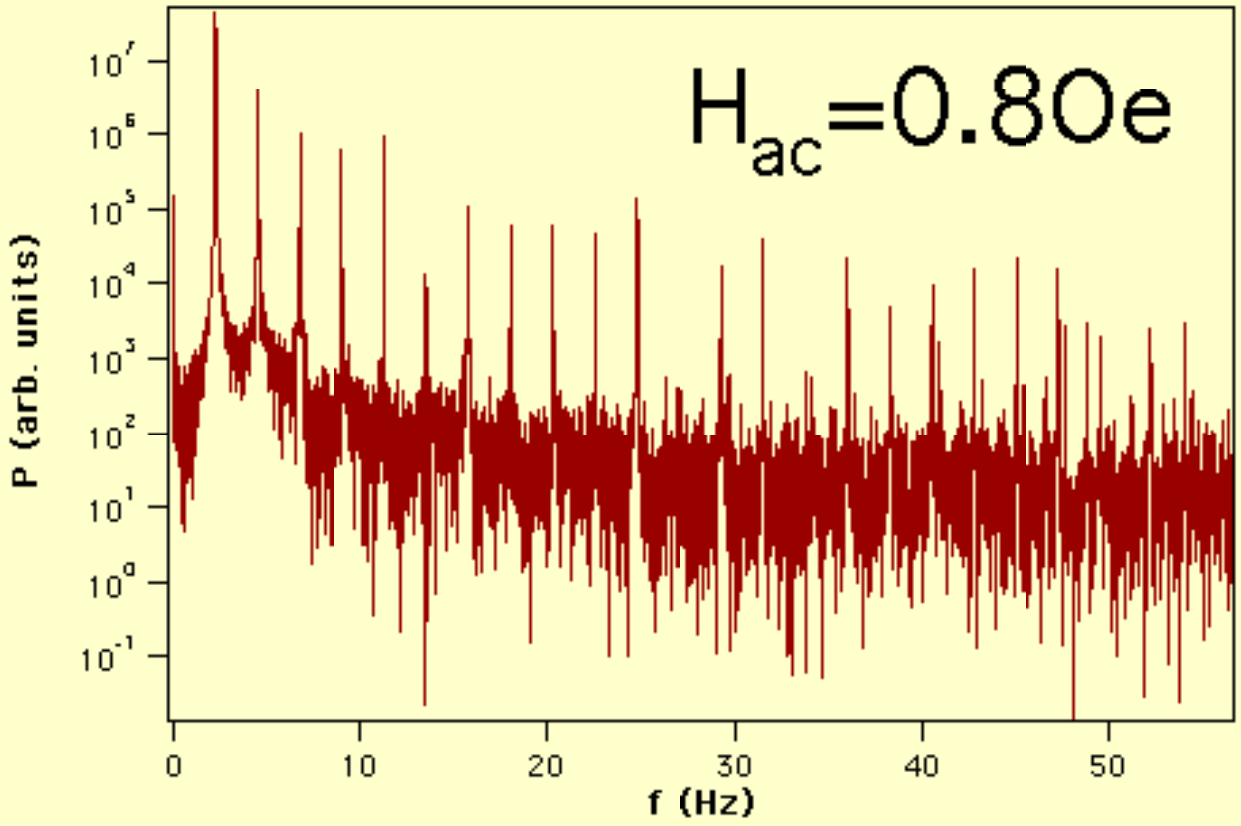


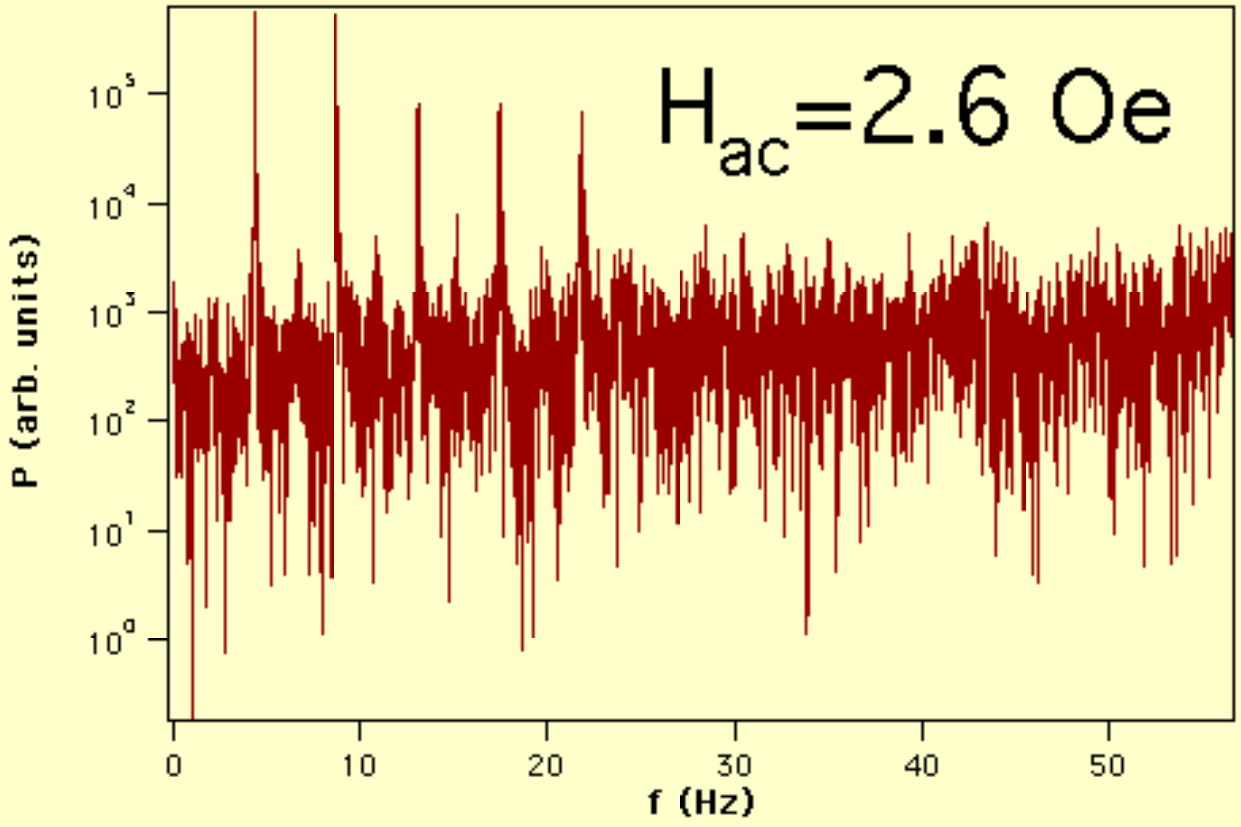


Power spectra

strobed detector signal

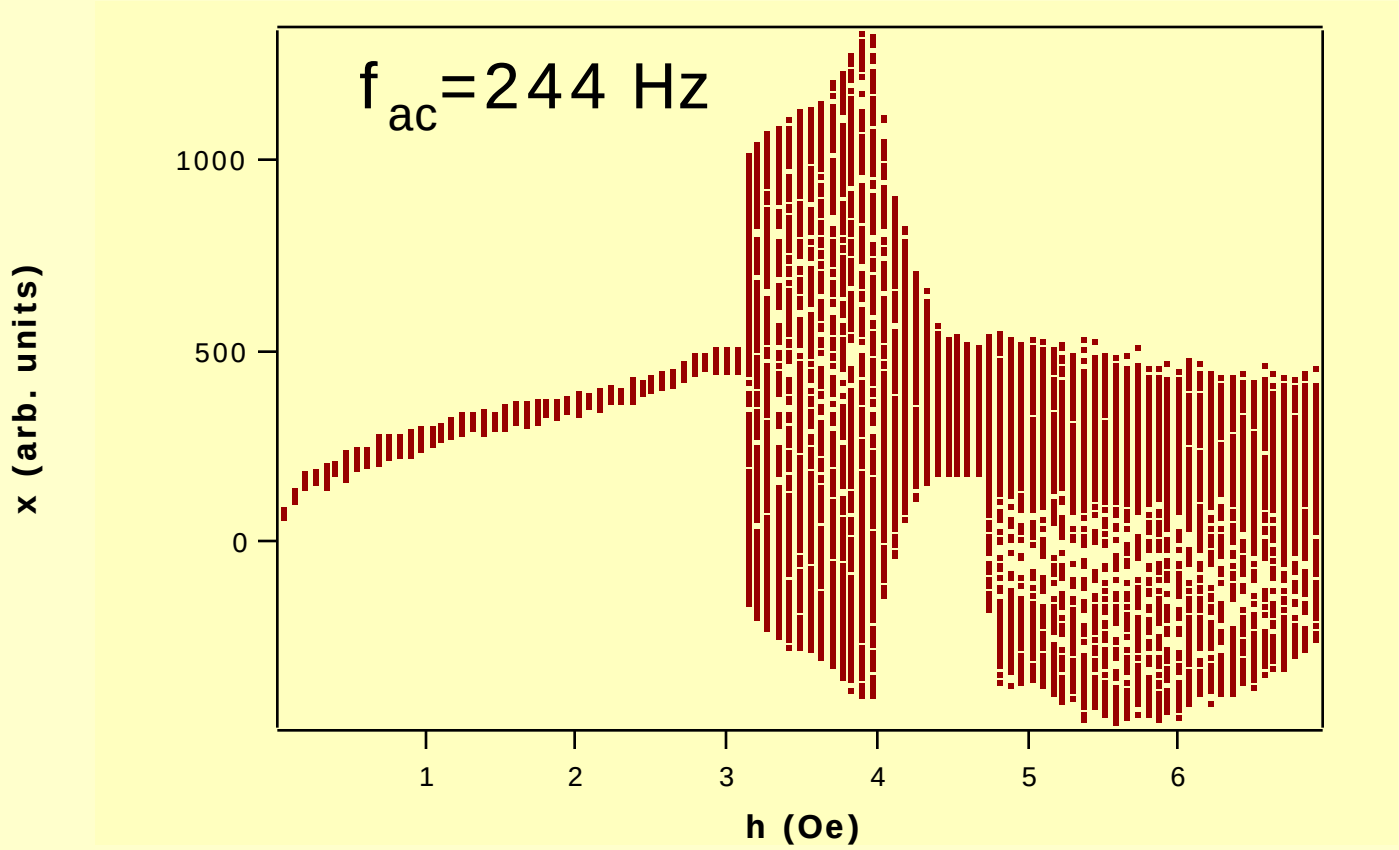
magnetostrictive ribbon driven at 114 Hz.





Bifurcation diagram

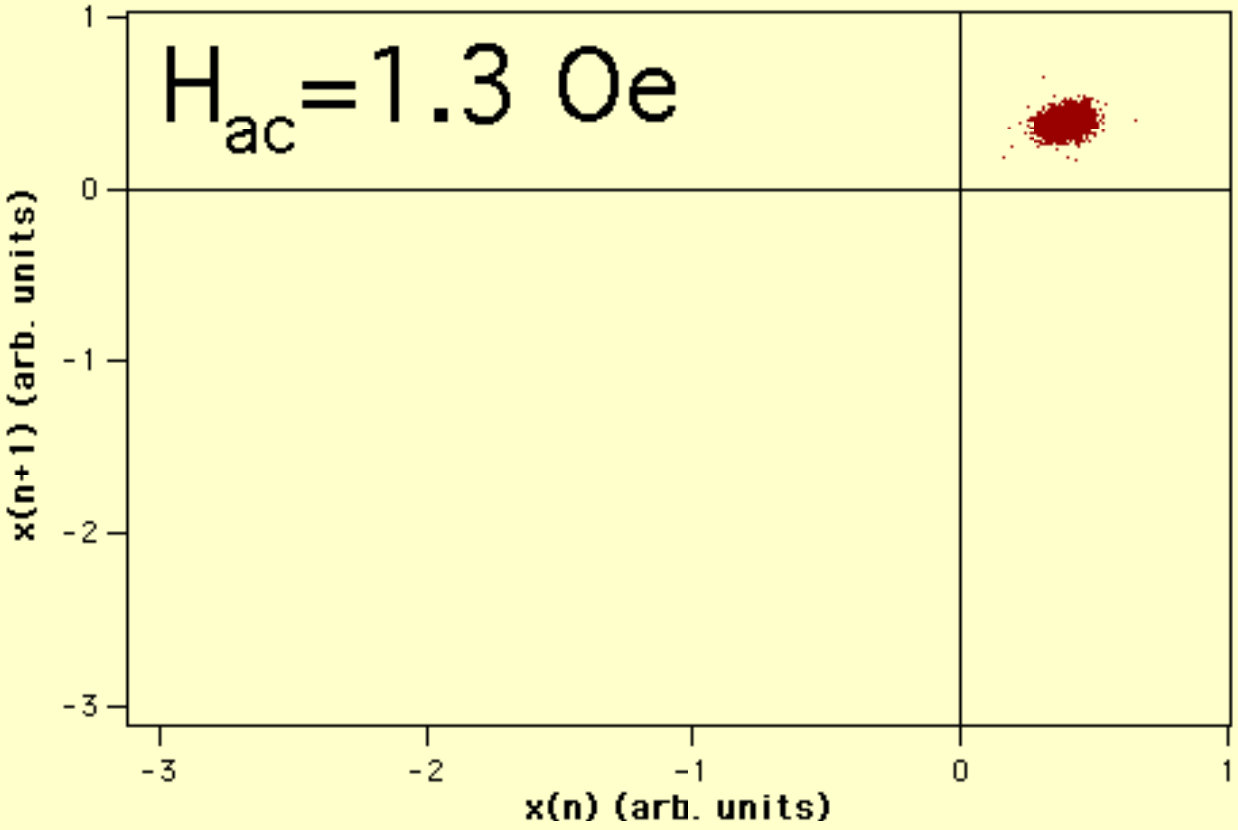
Magnetostrictive 2605sc ribbon
Driven at 244 Hz.

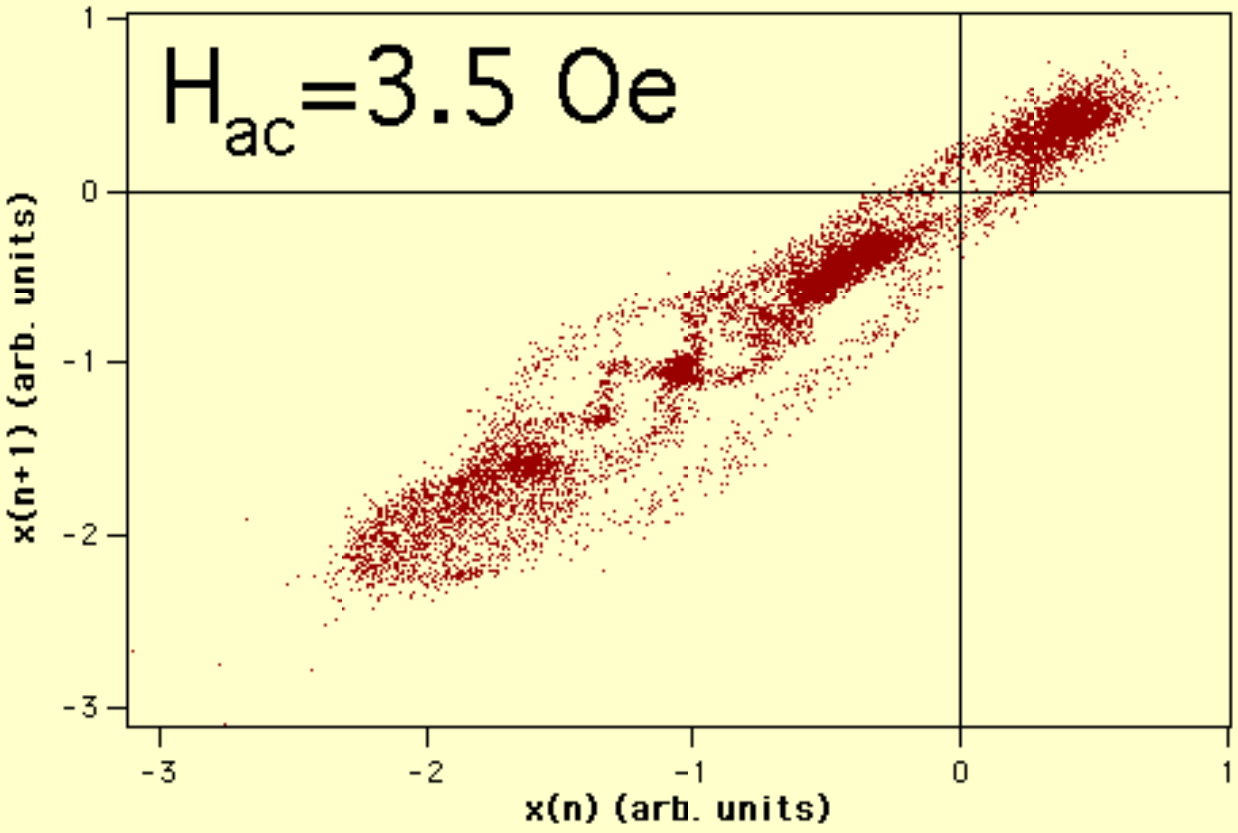


The peak-to-peak magnetic driving field is h,
while x is the value of the detector output when the
driving signal crosses zero in the positive direction.

Poincare Sections

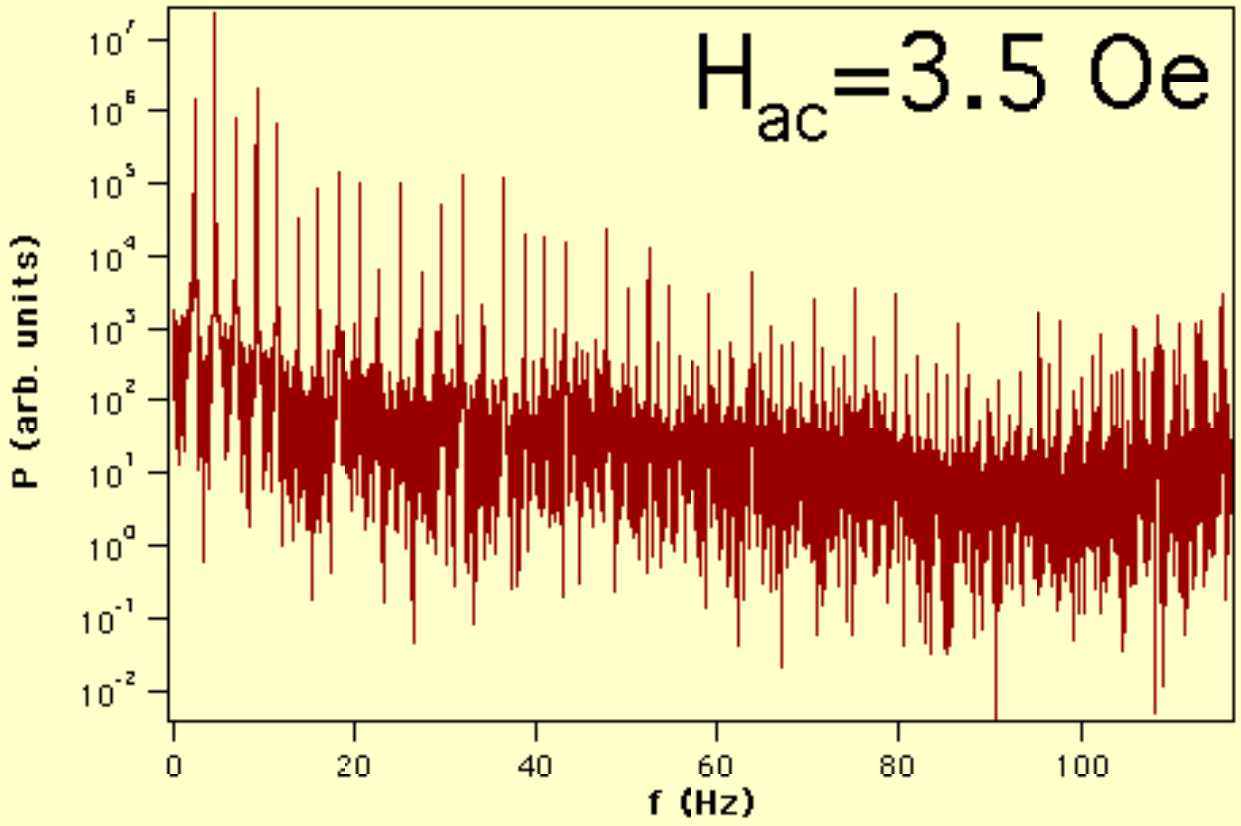
2605sc ribbon driven at 244 Hz.

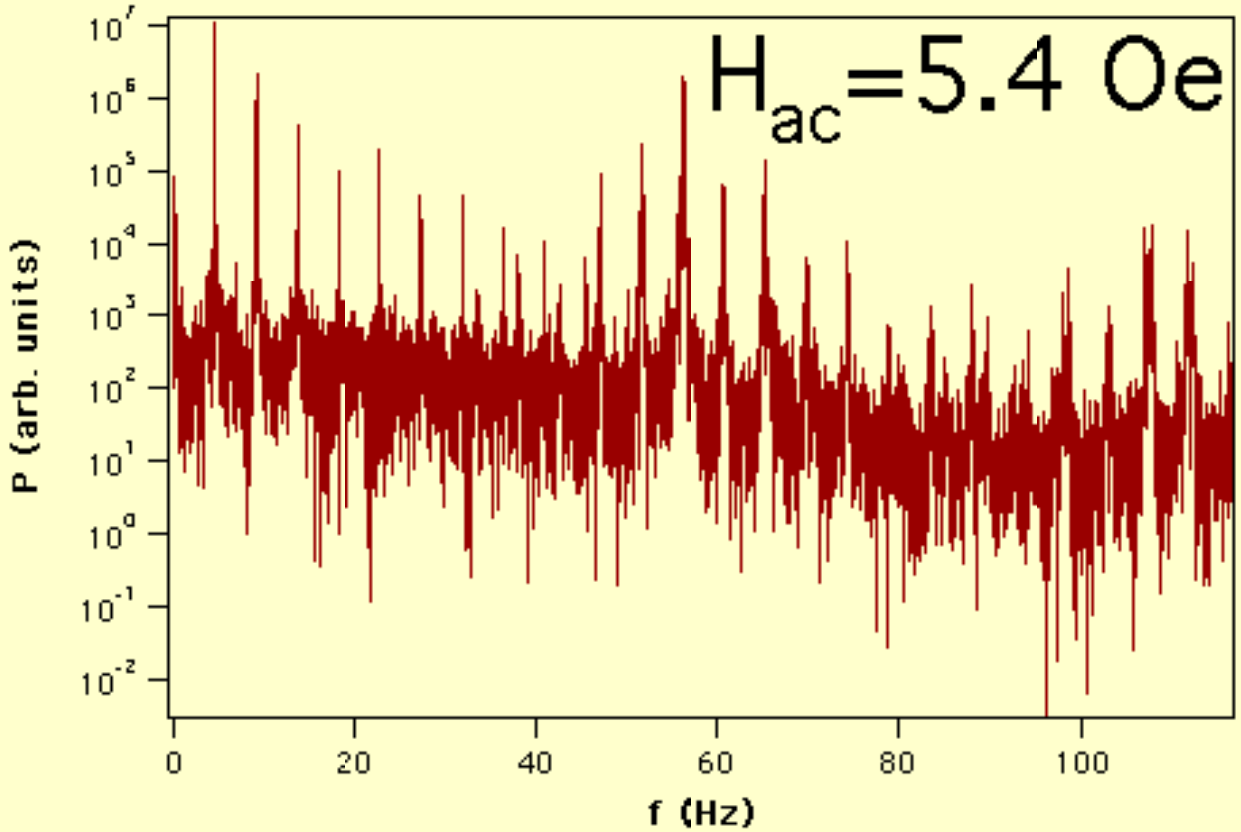




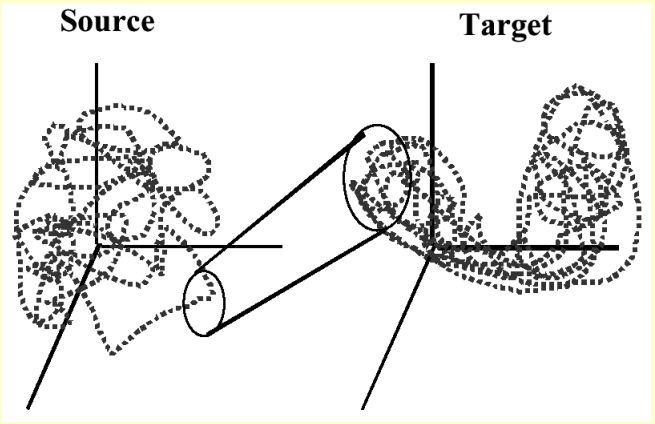
Power spectra

strobed detector signal
2605sc ribbon driven at 244 Hz.

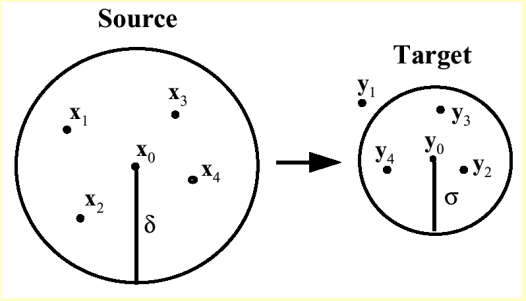




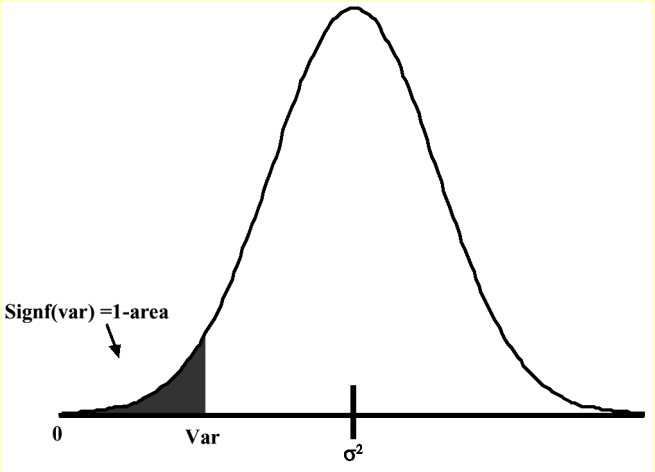
The Θ_{c^0} Statistic



- 2 Attractors, from embedding 2 simultaneous time series.
  - Designate: Source Attractor + Target Attractor
- Determination of length scale σ on Target Attractor
  - Select a point (a "center") y_0 on the target attractor.
  - Gather N nearest neighbors of this center, where N is large enough to achieve good statistics but small enough to calculate a minimum length scale.
  - Determine the variance of these points.
  - Set the significance of the variance equal 0.95 and solve for $\sigma(y_0)$.
  - Repeat for a number of other centers and average values to find attractor wide σ.

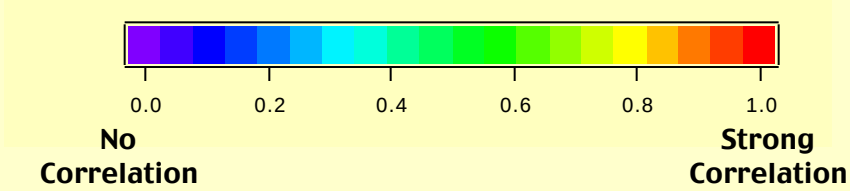


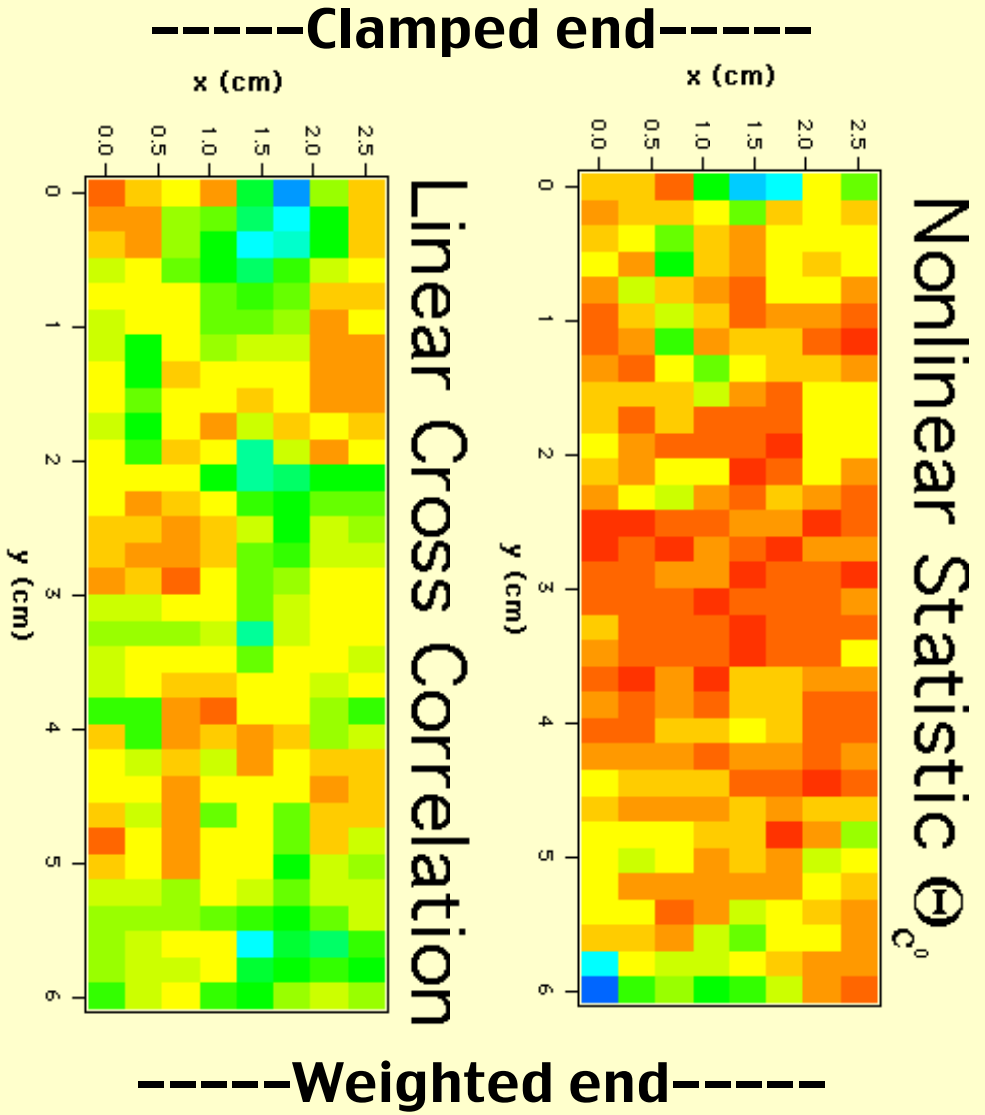
- Determination of Function Statistic
  - Select a center x_0 on source attractor.
  - Gather all of the points within some radius δ of this center.
  - Find all the simultaneous (in time) points on the target attractor.
  - Find the variance of these points and calculate the significance of this variance $\Theta_{c^0}(x_0)$.
  - Vary δ to maximize the significance for this point.
  - Repeat for a number of other centers and average to find a value for the function statistic.



Comparison Linear Cross Correlation & Non-linear Θ_{c^0}

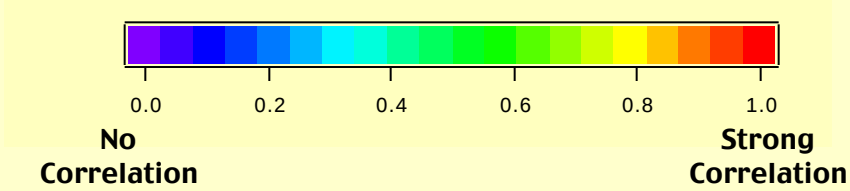
- Two Simultaneous Strobed Signals
- Magnetostrictive Ribbon
- Reference at center
- Drive f_o = 114 Hz
- H_{ac} = 1.4 Oe

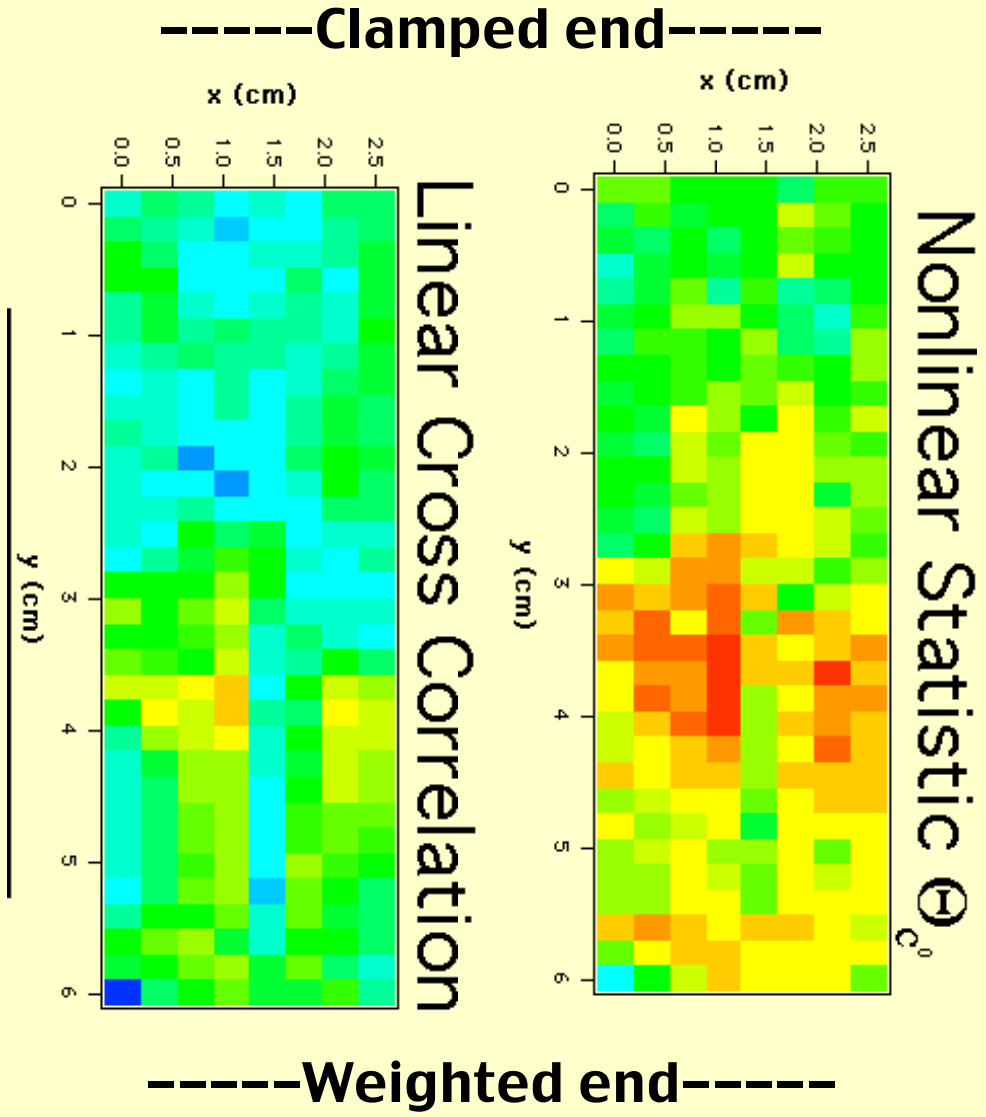




Comparison Linear Cross Correlation & Non-linear Θ_{C^0}

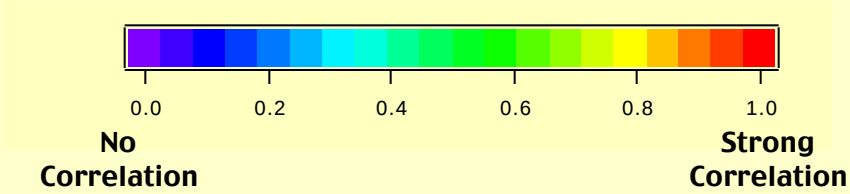
- Two Simultaneous Strobed Signals
- Magnetostrictive Ribbon
- Reference at center
- Drive f_o = 114 Hz
- H_{ac} = 6.1 Oe

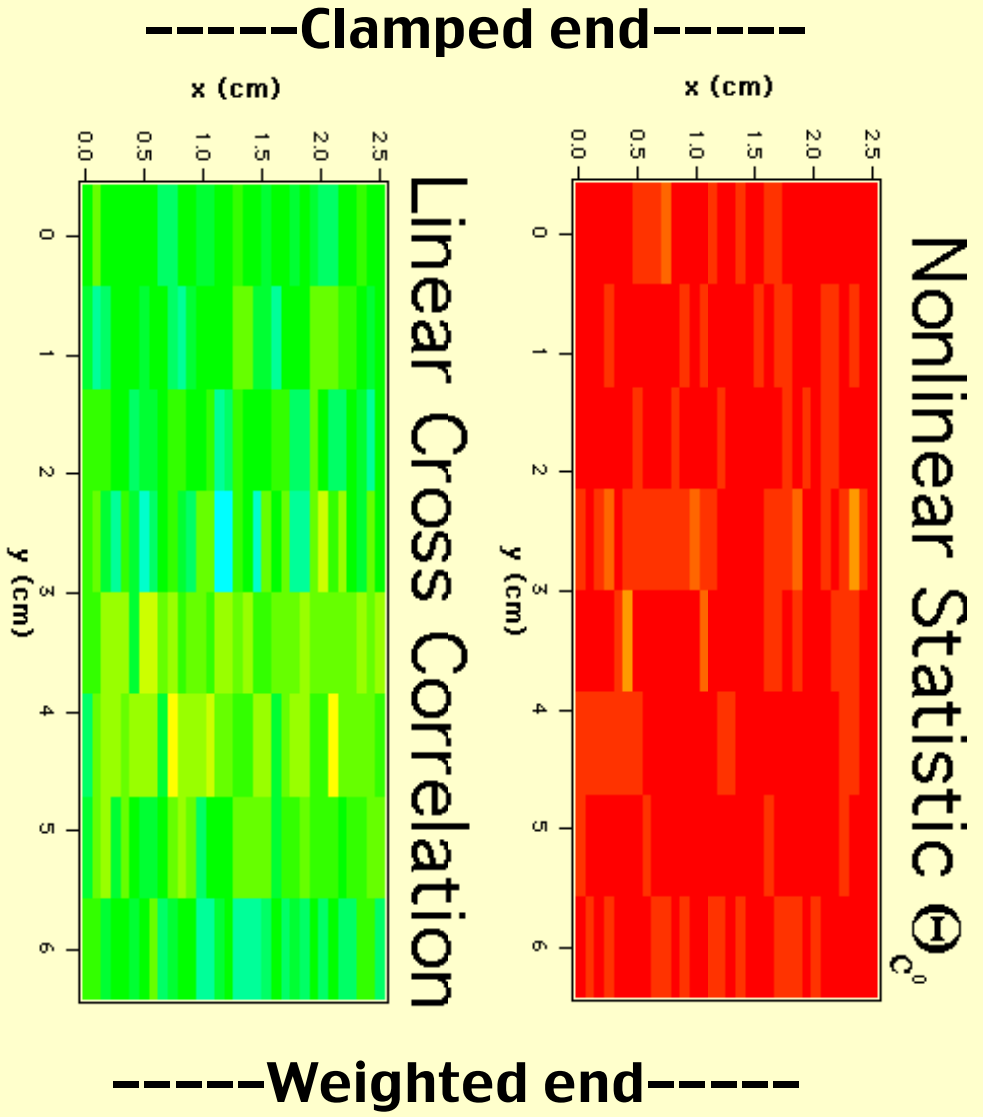




Comparison Linear Cross Correlation & Non-linear Θ_{C^0}

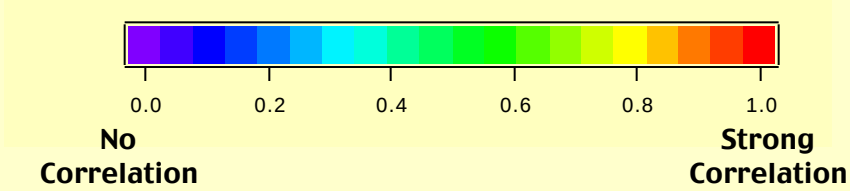
- Two Simultaneous Strobed Signals
- Magnetostrictive Ribbon
- Reference at center
- Drive f_o = 244 Hz
- H_{ac} = 3.4 Oe

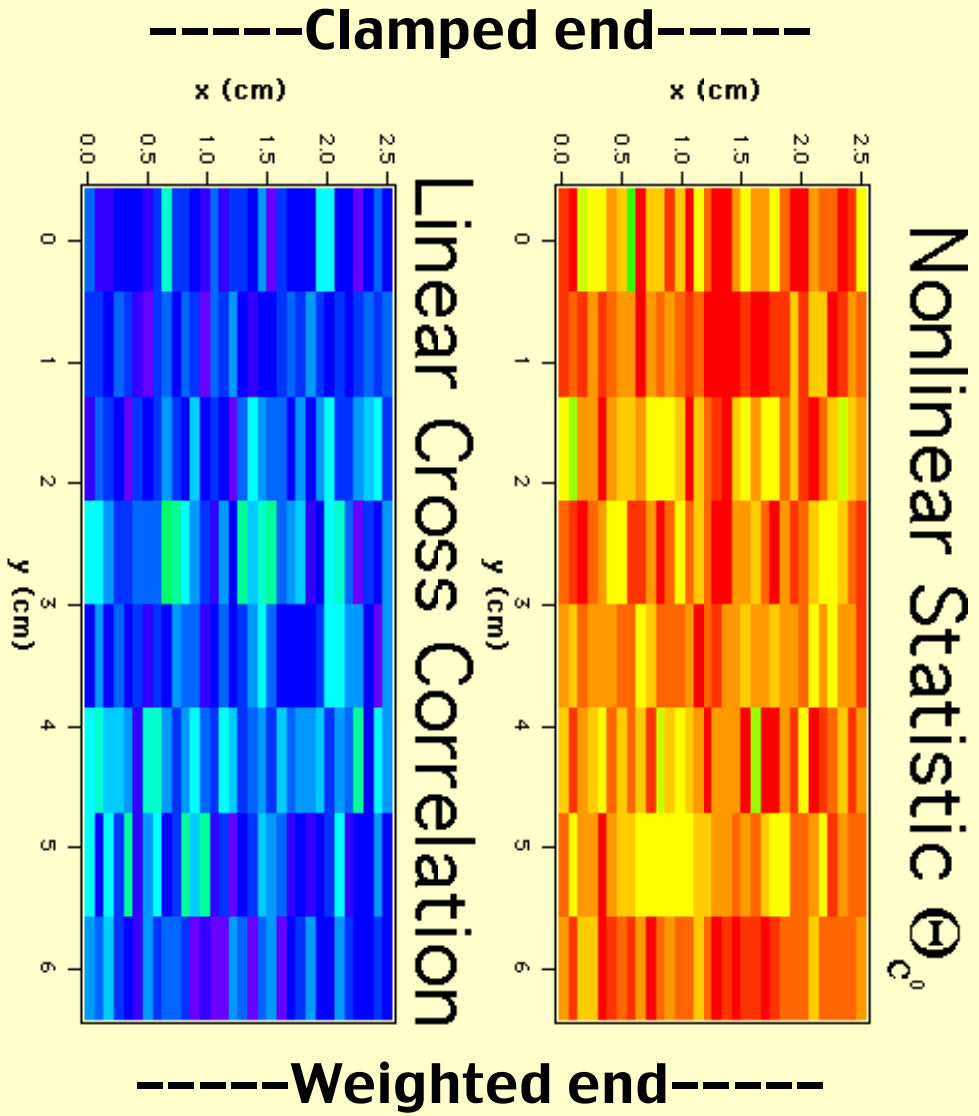




Comparison Linear Cross Correlation & Non-linear Θ_{c^0}

- Two Simultaneous Strobed Signals
- Magnetostrictive Ribbon
- Reference at center
- Drive f_o = 244 Hz
- H_{ac} = 6.0 Oe





Conclusions

- Magnetostrictive Metglas Ribbon Pendulum
  - Displays non-linear behavior
    - Quasiperiodicity (chaos)
    - Bifurcations as functions of drive amplitude.
- The non-linear Θ_{C^0} statistic
  - Shows the existance of a functional (but non-linear) spatial correlation across the magnetostrictive sample in the presence of noise.
- The linear cross correlation
  - Fails to show a strong spatial correlation.

Relevant Pre-Prints

- PDF Preprints are available at:

  http://chaos-mac.nrl.navy.mil/Section_Stuff/Recent_Pub.html

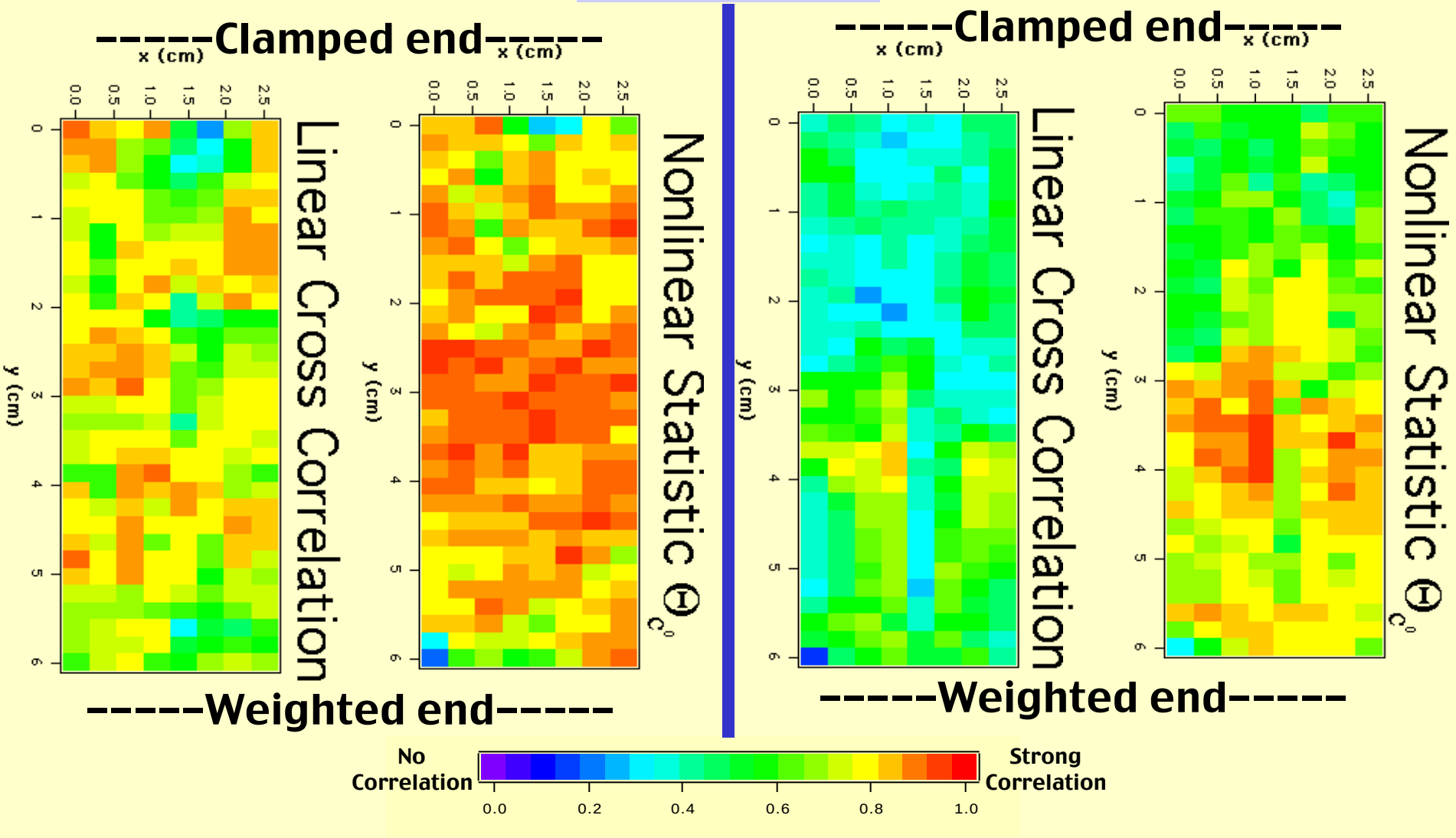
- These include:

  - Spatiotemporal Nonlinear Dynamics of a Magnetoelastic Ribbon, T. L. Carroll, M. D. Todd, F. J. Rachford, L. M. Pecora. (This paper)
  - Detecting Functional Relationships between Simultaneous Time Series, L. M. Pecora, C. L. Goodridge, T. L. Carroll, and F. J. Rachford.
    (The development of the Θ_{c0} statistic)
  - Functionality and Quasi-periodicity in Spatio-Temporal Dynamics of Yttrium Iron Garnet Films, C.L. Goodridge, F.J. Rachford, L.M. Pecora, and T.L. Carroll.
    (The use of Θ_{c0} in another spatio-temporal dynamical system.)

Statistic Comparison

Magnetostrictive Ribbon: Two Simultaneous Strobed Signals, Reference at center

H_{ac} = 1.4 Oe f_o = 114 Hz H_{ac} = 6.1 Oe



Statistic Comparison

Magnetostrictive Ribbon: Two Simultaneous Strobed Signals, Reference at center

H_{ac} = 3.4 Oe f_o = 244 Hz H_{ac} = 6.0 Oe